

Thermal phase transition in QCD ¹

A.V. Smilga

Department of Physics, SUNY at Stony Brook, Stony Brook, NY 11794-3800, USA ²

Abstract

We give a review of modern theoretical understanding of the problem of thermal phase transition in *QCD*. The existence or non-existence of such transition depends on the nature of the gauge group, the number of light quark flavors, and on the value of quark masses. Numerical lattice measurements indicate that the phase transition *does* not occur for the experimentally observed values of quark masses.

1 Introduction.

The properties of *QCD* medium at finite temperature have been the subject of intense study during the last 15 years. It was realized that the properties of the medium undergo a drastic change as the temperature increases. At low temperatures, the system presents a gas of colorless hadron states — the eigenstates of the *QCD* hamiltonian at zero temperature. When the temperature is small, this gas is composed mainly of pions — other mesons and baryons have higher mass and their admixture in the medium is exponentially small $\sim \exp\{-M/T\}$. At small temperature, also the pion density is small — the gas is rarefied and pions practically do not interact with each other.

However, when the temperature increases, pion density grows, the interaction becomes strong, and also other strongly interacting hadrons appear in the medium. For temperatures of order $T \sim 150$ Mev and higher, the interaction becomes so strong that the hadron states do not present a convenient basis to describes the properties of the medium anymore, and no analytic calculation is possible.

On the other hand, when the temperature is very high, much higher than the characteristic hadron scale $\mu_{hadr} \sim 0.5$ Gev, theoretical analysis becomes possible again. Only in this range, the proper basis are not hadron states but quarks and gluons — the elementary fields entering the *QCD* lagrangian. For high temperatures, a characteristic energy of quarks and gluons travelling through the medium is also high, the effective coupling constant is small, and the

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²Permanent address: ITEP, B. Cheremushkinskaya 25, Moscow 117259, Russia.

system presents the *quark-gluon plasma*. Its properties are in many respects very similar to the properties of the usual non-relativistic plasma involving charged particles with weak Coulomb interaction. The only difference is that quarks and gluons carry not the electric, but color charge. The perturbation theory in the coupling constant can be developed and many thermodynamic (such as free energy) and kinetic (such as viscosity) characteristics of the medium can be analytically evaluated [1].

Thus, the properties of the system at low and at high temperatures have nothing in common. A natural question arises: What is the nature of the transition from low-temperature hadron gas to high-temperature quark-gluon plasma? Is it a *phase* transition? If yes, what is its order? I want to emphasize that this question is highly non-trivial. A drastic change in the properties of the system in a certain temperature range does not guarantee the presence of the phase transition *point* where free energy of the system or its specific heat is discontinuous. Recall that there is no phase transition between ordinary gas and ordinary plasma.

We shall see in the following that, as far as the real *QCD* with particular values of quark masses is concerned, the answer is probably negative. What really happens is not the phase transition but a sharp crossover — "almost" a second-order phase transition. However, the real phase transition *does* occur in some relative theories — in pure Yang-Mills theory (when the quark masses are sent to infinity) and in *QCD* with 2 or 3 exactly massless quark flavors.

There are at least 4 reasons why this question is interesting to study:

1. It is just an amusing theoretical question.
2. Theoretic conclusions can be checked in lattice numerical experiments. Scores of papers devoted to lattice study of thermal properties of QCD have been published.
3. Perhaps, a direct experimental study would be possible on RHIC — high-energy ion collider which is now under construction. I'll discuss the possibility to observe a beautiful effect, the so called *disoriented chiral condensate* in the end of the lecture.
4. During the first second of its evolution, our Universe passed through the stage of high- T quark-gluon plasma which later cooled down to hadron gas (and eventually to dust and stars, of course). It is essential to understand whether the phase transition did occur at that time. A profound first-order phase transition would lead to observable effects. We know (or almost know — the discussion of this question has not yet completely died away) that there were no such transition. But it is important to understand why.

Note that there is also a related but *different* question — what are the properties of relatively cold but very dense matter and whether there is a phase transition when the chemical potential corresponding to the baryon charge rather than the temperature is increased. This lecture will be devoted exclusively to the thermal properties of QCD, and we shall assume zero baryon charge density.

2 Pure Yang-Mills theory: deconfinement phase transition.

This is the system where the phase transition from the glueball phase to the gluon plasma phase *does* occur. This result has been obtained long ago by Polyakov [2] and Susskind [3]. On the heuristic level, the reasoning is the following:

We know (for real QCD — from experiment, and for pure YM theory — from theoretical arguments and from lattice measurements) that the theory enjoys confinement at low temperature. That means that the potential between the test heavy quark and antiquark grows linearly at large distances:

$$T = 0 : \quad V_{Q\bar{Q}}(r) \sim \sigma r, \quad r \rightarrow \infty \quad (2.1)$$

On the other hand, at high temperature when the system presents a weakly interacting plasma of gluons, the behavior of the potential is quite different:

$$T \gg \mu_{had} : \quad V_{Q\bar{Q}}(r) \sim \frac{g^2(T)}{r} e^{-m_D r} \quad (2.2)$$

Here $m_D \sim gT$ is the Debye mass, and the potential is the Debye screened potential much similar to the usual Debye potential between static quarks in non-relativistic plasma. There is no confinement at large T . There should be some point T_c (the critical temperature) where the large r asymptotics of the potential changes and the phase transition from the confinement phase to the Debye screening phase occurs.

These simple arguments can be formulated in a rigorous way. Consider the partition function of the system written as the Euclidean path integral. It is known since Matsubara that, at finite T , the fields are defined on the cylinder: Euclidean time τ lies within the range $0 \leq \tau \leq \beta = 1/T$, and one should impose periodic boundary conditions on the gluon fields:

$$A_\mu^a(\mathbf{x}, \beta) = A_\mu^a(\mathbf{x}, 0) \quad (2.3)$$

Let us choose a gauge where A_0^a is time-independent. Introduce the quantity called the Polyakov loop

$$P(\mathbf{x}) = \frac{1}{N_c} \text{Tr} \exp\{ig\beta A_0^a(\mathbf{x})t^a\} \quad (2.4)$$

It is just a Wilson loop on the contour which winds around the cylinder. Consider the correlator

$$C_T(\mathbf{x}) = \langle P(\mathbf{x}) P^*(0) \rangle_T \quad (2.5)$$

One can show [4] that the correlator (2.5) is related to the free energy of the test heavy quark-antiquark pair immersed in the plasma.

$$C_T(\mathbf{x}) = \frac{3}{4} \exp\{-\beta F_{Q\bar{Q}}^{(3)}(r)\} + \frac{1}{4} \exp\{-\beta F_{Q\bar{Q}}^{(0)}(r)\} \quad (2.6)$$

where $r = |\mathbf{x}|$. $F_{Q\bar{Q}}^{(3)}(r)$ and $F_{Q\bar{Q}}^{(0)}(r)$ are free energies of test quark-antiquark pairs (alias static potentials) in the triplet and, correspondingly, the singlet net color state. Let us take now the limit $r \rightarrow \infty$. The quantity

$$C_T(\infty) = \lim_{r \rightarrow \infty} C_T(r) \quad (2.7)$$

plays the role of the *order parameter* of the deconfinement phase transition. At small T , $F_{Q\bar{Q}}(r)$ grow linearly at $r \rightarrow \infty$ and $C_T(\infty) = 0$. At large T , free energies do not grow and $C_T(\infty)$ is some non-zero constant (if one would naively substitute in Eq.(2.6) the Debye form of the potentials (2.2), one would get $C_{T \gg \mu_{hadr}}(\infty) = 1$, but it is not quite true because $F_{Q\bar{Q}}(r)$ involve also a constant depending on the ultraviolet cutoff of the theory. See [5, 6] for detailed discussion). There is a phase transition in between.

What are the properties of this phase transition ? There are not quite rigorous but suggestive theoretical arguments based on the notion of “universality class” [7] which predict different properties for different gauge groups. The main idea is that the pure YM theory based on $SU(2)$ color group has some common features with the Ising model (with global symmetry Z_2), the theory with $SU(3)$ gauge group — with a generalized Ising model (the Potts model) with the global symmetry Z_3 etc. The Ising model has the second order phase transition, and the same should be true for pure $SU(2)$ gauge theory. Systems with Z_N symmetry display, however, the first order phase transition, and the same should be true for pure $SU(N \geq 3)$ theory. The lattice data [8] are in a nice agreement with this prediction. Also, critical indices of the second order phase transition were measured. Their numerical values are close to the numerical values of critical indices in the Ising model.

2.1 Bubble confusion.

There was a long-standing confusion concerning the nature of deconfinement phase transition in pure YM theory. It has been clarified only recently and I want to dwell on this question in more details.

In scores of papers published since 1978, it was explicitly or implicitly assumed that one can use the cluster decomposition for the correlator (2.5) at large T and attribute the meaning to the temperature average $\langle P \rangle_T$. Under this assumption, the phase of this average can acquire N_c different values: $\langle P \rangle_T = C \exp\{2\pi i k / N_c\}$, $k = 0, 1, \dots, N_c - 1$ which would correspond to N_c distinct physical phases and to the spontaneous breaking of the discrete Z_N -symmetry. In recent [9], the surface energy density of the domain walls separating these phases has been evaluated.

However, the standard interpretation is wrong. In particular:

1. Only the correlator (2.5) has the physical meaning. The phase of the expectation value $\langle P \rangle_T$ is not a physically measurable quantity. There is only *one* physical phase in the hot YM system.

2. The “walls” found in [9] should not be interpreted as physical objects living in Minkowski space, but rather as Euclidean field configurations, kind of “planar instantons” appearing due to non-trivial $\pi_1(\mathcal{G}) = \mathcal{Z}_N$ where $\mathcal{G} = SU(N)/Z_N$ is the true gauge symmetry group of the *pure* Yang-Mills system.
3. The whole bunch of arguments which is usually applied to non-abelian gauge theories can be transferred with a little change to hot *QED*. The latter also involves planar instantons appearing due to non-trivial $\pi_1[U(1)] = Z$. These instantons should *not* be interpreted as Minkowski space walls.

It is impossible to present an adequate discussion of this issue in this short lecture. We refer the reader to [6] where such discussion was given. Here we restrict ourselves by outlining some heuristic physical arguments.

Right from the beginning, one meets a puzzle. In standard approach, Z_N is broken spontaneously at *high* temperature and restored at *low* temperature. This is very strange and unusual. The opposite is much more common in physics. There are some models where symmetry breaking survives and can even be induced at high temperature [10], but the mechanism of this breaking is quite different from what can possibly occur in the pure YM theory.

The second observation is that Z_N symmetry which is presumably broken in the deconfinement phase is just not there in the continuum theory: the gluon fields are not transformed under the action of the center. Z_N - symmetry is present in the standard lattice version of the theory but is absent, again, in the lattice theory involving adjoint matrices $O^{ab} = \text{Tr}\{t^a U t^b U^\dagger\}$ rather than the unitary matrices U .

It was mentioned earlier that the Polyakov loop expectation value $\langle P \rangle_T$ as such has no physical meaning. Let us explain why. Being taken at face value, $\langle P \rangle_T$ would measure the free energy of a single fundamental heavy source immersed in the system [11]: $\langle P \rangle_T = \exp\{-\beta F_T\}$. As a matter of fact, nonzero phase of $\langle P \rangle_T$ would correspond to the complex free energy which is an obvious nonsense. But the point is that one just cannot put a single fundamental source into the system due to the Gauss law constraint [12]. The net color charge should be zero, and a fundamental source cannot be screened by gluons — the only dynamic fields in the lagrangian of the theory and in the heat bath ³

What one can well do is to immerse a heavy quark-antiquark pair and measure thereby the correlator (2.5) which *is* physical. Or, say, for the $SU(3)$ gauge group, one can immerse 3 heavy quarks at different points and measure $\langle P(\mathbf{x})P(\mathbf{y})P(0) \rangle_T$ which is physical, again, but, in contrast to $\langle P \rangle_T$, does not involve the phase uncertainty.

Actually, the delusion of spontaneous Z_N breaking in pure YM theory persisted for so long

³To be quite precise, the net color charge can be made non-zero due to a boundary term at spatial infinity. In a finite spatial box, such a term can appear when non-standard (non-periodic) boundary conditions are chosen. But then the phase of $\langle P \rangle_T$ would be exactly determined by these boundary conditions. As the physical properties of the theory cannot depend on b.c. when the box is large enough, it is just another way to say that the phase is not physical.

because people habitually described this system in terms of $A_0^a(\mathbf{x})$ (more exactly — in terms of $\Omega_{\mathbf{x}} = \exp\{i\beta g A_0^a(\mathbf{x})t^a\}$) which are not the dynamic variables entering the hamiltonian but the variables dual to the Gauss law constraints. A close analogy can be drawn with the Ising model in 2 dimensions. When expressed in terms of the original spin variables σ_i , the system is ordered at low temperatures and disordered at high temperatures — the spontaneously broken Z_2 - symmetry is restored there. One can make, however, the Kramers-Wannier transformation and describe the system in terms of the dual or "disorder" variables η_i [13]. When the normal temperature is high, the dual temperature is low, and the dual hamiltonian $H^*[\eta_i]$ describes, indeed, the system where Z_2 symmetry is broken at high temperatures and restored at low temperatures. But the variables η_i are not physical observables and, as far as any Gedanken physical experiment is concerned, Z_2 - symmetry in the Ising model *is* restored at high temperatures.

To summarize, there is only one physical phase at high T . Its properties are relatively simple — it is the weakly interacting plasma of gluons. The description in terms of dual variables is useful for some purposes (e.g. the universality class arguments of Ref. [7] which predict the order of the phase transition are based on the dual description), but one should be very careful not to read out in it something which is not in Nature.

3 QCD with massless quarks.

If the theory involves besides gluons also quarks with finite mass, the static interquark potential $V_{Q\bar{Q}}(r)$ does not grow at large distances anymore even at $T = 0$. Dynamic quarks screen the potential of static sources. One can visualize this screening thinking of the color gluon tube stretched between two static fundamental sources being torn apart in the middle with the formation of an extra quark-antiquark pair. Thus, in QCD with quarks, the Wilson loop average has the perimeter rather than the area law ⁴. The correlator of two Polyakov loops (2.5) tends to a constant at large distances universally at low and at high temperature, and this correlator cannot play the role of the order parameter of phase transition.

Still, the phase transition can occur and does occur in some versions of the theory. It is associated, however, not with change in behavior of the correlator (2.5), but with *restoration of chiral symmetry* which is spontaneously broken at zero temperature.

Consider YM theory with $SU(3)$ color group and involving N_f massless Dirac fermions in the fundamental representation of the group. The fermion part of the lagrangian is

$$L_f = i \sum_f \bar{q}_f \gamma_\mu \mathcal{D}_\mu q_f \quad (3.1)$$

where $\mathcal{D}_\mu = \partial_\mu - igA_\mu^a t^a$ is the covariant derivative. The lagrangian (3.1) is invariant under

⁴That does not mean that there is no confinement — as earlier, only the colorless states are present in the physical spectrum. But the behavior of the Wilson loop is not a good signature of confinement anymore.

chiral transformations of fermion fields:

$$q_{L,R} \rightarrow A_{L,R} q_{L,R} \quad (3.2)$$

where $q_{L,R} = \frac{1}{2}(1 \pm \gamma^5)q$ is the flavor vector with N_f components and $A_{L,R}$ are two different $U(N_f)$ matrices. Thus, the symmetry of the classical lagrangian is $U_L(N_f) \otimes U_R(N_f)$. Not all Nöther currents corresponding to this symmetry are conserved in the full quantum theory. It is well known that the divergence of the singlet axial current $j_\mu^5 = \sum_f \bar{q}_f \gamma_\mu \gamma^5 q_f$ is non-zero due to anomaly:

$$\partial_\mu j_\mu^5 \sim g^2 \epsilon^{\mu\nu\alpha\beta} G_{\mu\nu}^a G_{\alpha\beta}^a \quad (3.3)$$

Thus, the symmetry of quantum theory is $SU_L(N_f) \otimes SU_R(N_f) \otimes U_V(1)$. It is the experimental fact that (for $N_f = 2, 3$, at least) this symmetry is broken spontaneously down to $U_V(N_f)$. The order parameter of this breaking is the chiral quark condensate $\langle \sum_f \bar{q}_f q_f \rangle_0$. This spontaneous breaking leads to appearance of the octet of pseudoscalar Goldstone states in the spectrum. Of course, in the real *QCD* the quarks are not exactly massless, the mass term is not invariant with respect to the symmetry (3.2) but only under $U_V(N_f)$. As a result, in real World we have the octet of light (but not massless) pseudo- Goldstone pseudoscalar states (π, K, η). But the small mass of pseudogoldstones and the large splitting between the massive states of opposite parity (ρ/A_1 , etc.) indicate beyond reasonable doubts that the exact chiral symmetry (3.2) would be broken spontaneously in the massless case. As the masses of the strange and, especially, of u - and d - quarks are small [14], the mass term in the lagrangian can be treated as perturbation. E.g. the pion mass satisfies the relation

$$F_\pi^2 m_\pi^2 = (m_u + m_d) | \langle \bar{u}u \rangle_0 | \quad (3.4)$$

($F_\pi = 93$ Mev is the pion decay constant) and turns to zero in the chiral limit $m_{u,d} \rightarrow 0$.

It is noteworthy that the symmetry breaking pattern

$$SU_L(N_f) \otimes SU_R(N_f) \rightarrow SU_V(N_f) \quad (3.5)$$

depends crucially on the assumption that the gauge group involves at least 3 colors. For $SU(2)$ color group where quarks and antiquarks belong to the same representation (the fundamental representation of the $SU(2)$ group is pseudoreal: $\mathbf{2} \equiv \bar{\mathbf{2}}$), the symmetry group of the lagrangian (3.1) is much higher. It is $U(2N_f)$ and involves also mixing between quarks and antiquarks. $U_A(1)$ - part of this symmetry is anomalous and the formation of chiral condensate breaks spontaneously the remaining $SU(2N_f)$ down to a symplectic group [15]:

$$SU(2N_f) \rightarrow Sp(2N_f) \quad (3.6)$$

As a result, $2N_f^2 - N_f - 1$ Goldstone bosons living on the coset space appear. For $N_f = 2$, we have not 3 as usual, but 5 “pions”. This fact is important to understand for people who would wish to study numerically on lattices the spontaneous chiral symmetry breaking with $SU(2)$ gauge group.

As far as the thermal properties of the theory are concerned, the point is that a spontaneously broken symmetry must be restored under a sufficient heating. There should be a critical temperature T_c above which the fermion condensate $\langle \bar{q}q \rangle_T$ is zero. This is the temperature of phase transition and $\langle \bar{q}q \rangle_T$ is the order parameter associated with the transition.

Note that the phenomenon of spontaneous chiral symmetry breaking is specific for theories with *several* light quark flavors. In the theory with $N_f = 1$, the non-anomalous part of the symmetry of the lagrangian is just $U_V(1)$. It stays intact after adding the mass term and after taking into account the formation of the condensate $\langle \bar{q}q \rangle$. The condensate is still formed, but it does not correspond to spontaneous breaking of any symmetry and need not vanish at high temperature. So, it does not. At high temperatures when the effective coupling is small, it can be evaluated semiclassically in the instanton approach [16, 17], and one can show that it falls down as a power of temperature and never reaches zero.

There is *no* phase transition in *QCD* with only one light or massless flavor. But it *does* occur when the number of massless flavors N_f is 2 or more.

The melting down of quark condensate can be studied analytically at low temperature when the medium presents a rarefied weakly interacting gas of pions with low energies. Their properties are described by the effective chiral lagrangian

$$\mathcal{L} = \frac{1}{4}F_\pi^2 \text{Tr}\{\partial_\mu U \partial_\mu U^\dagger\} + \dots \quad (3.7)$$

where U is the $SU(N_f)$ matrix and the dots stand for higher derivative terms and the terms involving quark masses. When the characteristic energy and the quark masses are small, the effects due to these terms are suppressed and a perturbation theory (the *chiral perturbation theory* [18]) can be developed. In [19], the temperature dependence of $\langle \bar{q}q \rangle_T$ has been determined on the 3-loop level. In the approximation where only the presence of pions in the heat bath is taken into account and the effects due to non-zero m_u and m_d are neglected, the result has a rather simple form

$$\langle \bar{q}q \rangle_T = \langle \bar{q}q \rangle_0 \left[1 - \frac{T^2}{8F_\pi^2} - \frac{T^4}{384F_\pi^4} - \frac{T^6}{288F_\pi^6} \ln \frac{\Lambda}{T} + \dots \right] \quad (3.8)$$

The constant Λ depends on the higher-derivative terms in the effective lagrangian and can be fixed from experiment: $\Lambda \sim 500 \pm 100$ Mev. The dependence (3.8) together with the curves where only the 1 loop correction $\propto T^2$ and 2 loop correction $\propto T^4$ are taken into account (please, do not put attention to the “technical” curve marked $a_2^0 = 0$) is drawn in Fig. 1 taken from Ref. [19].

The expansion in the parameter $\sim T^2/8F_\pi^2$ makes sense when this parameter is small, i.e. when $T \leq 100 - 150$ Mev. Strictly speaking, one cannot extrapolate the dependence (3.8) for larger temperatures, especially having in mind that, at $T > 150$ Mev, the heat bath includes a considerable fraction of other than pion hadron states. But as we know anyhow that the phase transition with restoration of chiral symmetry should occur, the estimate of the phase transition temperature (i.e. the temperature when $\langle \bar{q}q \rangle_T$ hits zero) based on such an extrapolation is not altogether stupid. This estimate is

$$T_c \approx 190 \text{ Mev} \quad (3.9)$$

(A more accurate treatment which takes into account non-zero $m_{u,d}$ and also the presence of other mesons in the heat bath gives practically the same estimate as these two effects push T_c in opposite directions and practically cancel each other.)

4 Properties of phase transition. The real World.

Making the estimate (3.9), we tacitly assumed that the phase transition is of the second order: only the derivative $\partial \langle \bar{q}q \rangle_T / \partial T$ but not $\langle \bar{q}q \rangle_T$ is discontinuous at the phase transition point. Let us discuss the question whether this assumption is valid and under what conditions.

On the theoretical side, the situation is similar to that in pure YM case: not rigorous but suggestive arguments exist indicating that the phase transition is of the second order for 2 massless flavors. When $N_f \geq 3$, the phase transition is probably of the first order. The arguments are the following [20]:

The starting point is the observation that, in theories involving *scalar* fields, phase transition of the first order often occurs when the potential involves a cubic in fields term. One can recall in the first place a cubic Van-der-Vaals curve $P(\rho, T)$ which describe the first order water \leftrightarrow vapor phase transition. The simplest field theory example is the theory of real scalar field with the potential

$$V(\phi) = \lambda(\phi^2 - v^2)^2 - \mu\phi^3 \quad (4.1)$$

Assume for simplicity $\mu \ll \lambda v$. At $T = 0$, the potential has one global minimum at $\phi \approx v + 3\mu/8\lambda$. At non-zero temperature, the term $\sim \lambda T^2 \phi^2$ is added to the effective potential. At high temperature $T \gg v$, the minimum occurs at $\phi = 0$. One can be easily convinced that a *local* minimum at $\phi = 0$ appears at some temperature T^* when the local minimum at positive ϕ still exists. The latter disappears at some larger temperature T^{**} . In a certain temperature range, two minima of the potential, the old and the new one, coexist, one being a metastable state with respect to the other. This is exactly the physical situation of the first order phase transition.

Let us go back to *QCD*. A direct application of this reasoning is not possible because the *QCD* lagrangian does not involve scalar fields. The effective chiral lagrangian (3.7) is also of no immediate use because higher-derivative terms which stand for dots cannot be neglected

in the region close to critical temperature. Suppose, however, that in the region $T \sim T_c$ some other effective lagrangian in Ginzburg-Landau spirit can be written which depends on the composite colorless fields

$$\Phi_{ff'} = \bar{q}_{Rf} q_{Lf'} \quad (4.2)$$

A general form of the effective potential which is invariant under $SU_L(N_f) \otimes SU_R(N_f)$ is

$$V[\Phi] \sim g_1 \text{Tr}\{\Phi\Phi^\dagger\} + g_2 (\text{Tr}\{\Phi\Phi^\dagger\})^2 + g_3 \text{Tr}\{\Phi\Phi^\dagger\Phi\Phi^\dagger\} + g_4 (\det \Phi + \det \Phi^\dagger) + \dots \quad (4.3)$$

(the coefficients may be smooth functions of T). Now look at the determinant term. For $N_f = 2$, it is quadratic in fields while, for $N_f = 3$, it is cubic in fields and the effective potential acquires the structure similar to Eq.(4.1) which is characteristic for the systems with first order phase transition. A more refined analysis [20] shows that the first order phase transition is allowed also for $N_f \geq 4$, but not for $N_f = 2$ where the phase transition is of the second order.

It is even possible to argue that, for $N_f \geq 3$, one has not one but *two* phase transitions. The argument is based on the exact relation for the spectral density of Euclidean Dirac operator $\rho(\lambda)$ in zero-temperature QCD with N_f massless flavours at small but nonzero λ . It is possible to show that it involves a non-analytic term in λ at $N_f \geq 3$ [21]:

$$\rho(\lambda) = \frac{\Sigma}{\pi} + \frac{\Sigma^2(N_f^2 - 4)}{32\pi^2 N_f F_\pi^4} |\lambda| + o(\lambda^2) \quad (4.4)$$

where $\Sigma = | \langle \bar{q}q \rangle_0 |$. At nonzero temperature, both $\rho(0)$ (i.e. the chiral condensate) and the coefficient of $|\lambda|$ in $\rho(\lambda)$ are changed. Eventually (at $T \gg \mu_{hadr}$) they should vanish. One can imagine a situation when the chiral condensate turns to zero before the slope does. Or other way round.

If lattice people would eventually observe two temperature phase transitions in QCD with 3 massless quark flavors, I would be happy, of course. However, a canonical viewpoint that there is only one transition but of the first order may also be true. Existing lattice measurements favour this possibility.

Up to now, we discussed only pure YM theory and QCD with massless quarks. But the quarks have non-zero masses: $m_u \approx 4$ Mev, $m_d \approx 7$ Mev, and $m_s \approx 150$ Mev [14]. The question arises whether the non-zero masses affect the conclusion on the existence or non-existence and the properties of the phase transition.

The experimental (i.e. lattice) answer to this question appears to be positive [22]. In Fig. 2, a phase diagram of QCD with different values of quark masses m_s and $m_u = m_d$ is plotted.

Let us discuss different regions on this plot. When the quark masses are large, quarks effectively decouple and we have pure YM theory with $SU(3)$ gauge group where the phase transition is of the first order. When all the quark masses are zero, the phase transition is also of the first order. When masses are shifted from zero a little bit, we still have a first order phase transition because a finite discontinuity in energy and other thermodynamic quantities cannot disappear at once when external parameters (the quark masses) are smoothly changed.

But when all the masses are non-zero and neither are too small nor too large, phase transition is absent. Notice the bold vertical line on the left. When $m_u = m_d = 0$ and m_s is not too small, we have effectively the theory with two massless quarks and the phase transition is of the second order. The experimental values of quark masses (the dashed circle in Fig. 2) lie close to this line of second order phase transitions but in the region where no phase transition occurs. It is the experimental fact as measured in Ref. [22].

This statement conforms nicely with a semi-phenomenological theoretical argument of ref. [23] which displays that even *if* the first order phase transition occurs in QCD, it is rather weak. The argument is based on a generalized Clausius-Clapeyron relation. In college physics, it is the relation connecting the discontinuity in free energy at the first-order phase transition point with the sensitivity of the critical temperature to pressure. The Clausius-Clapeyron relation in *QCD* reads

$$\text{disc} < \bar{q}q >_T = \frac{1}{T_c} \frac{\partial T_c}{\partial m_q} \text{disc} \epsilon \quad (4.5)$$

where $\text{disc} \epsilon$ is the latent heat. The derivative $\frac{\partial T_c}{\partial m_q}$ can be estimated from theoretical and experimental information of how other essential properties of *QCD* depend on m_q and from the calculation of T - dependence of condensate at low temperature in the framework of chiral perturbation theory (see Fig.1 and the discussion thereof). The dependence on quark masses is not too weak. From that, assuming that the discontinuity in quark condensate is as large as $< \bar{q}q >_0$ (which is not true, of course), we get an estimate

$$\text{disc} \epsilon < 0.4 \text{ GeV/fm}^3$$

which is rather small compared to the characteristic free energy density of the system in the vicinity of $T_c \sim 190\text{MeV}$.

Thus, latent heat of the first order phase transition (assuming it is there) must be small which means that the phase transition is likely to disappear under a relatively small perturbation due to nonzero m_s .

The question is not yet completely resolved, and independent lattice measurements are highly desirable. Most probable is, however, that, when temperature is changed, hadron gas goes over to quark-gluon plasma and other way round without any phase transition. There is, however, a sharp crossover in a narrow temperature range which is similar in properties to a second-order phase transition (the “phase crossover” if you will).

5 Instantons and percolation.

In the analysis in previous two sections, we relied on the fact that chiral symmetry *is* broken at zero temperature. It is an experimental fact in real *QCD*, but it is important to understand from pure theoretical premises *why* it is broken and what is the mechanism of its restoration at higher temperatures.

A completely satisfactory answer to this question has not yet been obtained. The problem is that *QCD* at zero temperature is a theory with strong coupling and it is very difficult (may be impossible) to study the structure of *QCD* vacuum state analytically. However, a rather appealing qualitative physical picture exists which is based on the model of instanton-antiinstanton liquid and on the analogy with the so called percolation phase transition in doped superconductors [24]. We refer the reader to the Shuryak's book for the detailed discussion and elucidate here only crucial points of the reasoning.

The starting point is the famous Banks and Casher relation [25] connecting quark condensate to the mean spectral density of Euclidean Dirac operator $\rho(\lambda)$ at $\lambda \sim 0$. Let us explain how it is derived. Consider the Euclidean fermion Green's function $\langle q(x)\bar{q}(y) \rangle$ in a particular gauge field background. Introduce a finite Euclidean volume V to regularize theory in the infrared. Then the spectrum of massless Dirac operator is discrete and enjoys the chiral symmetry: for any eigenfunction $\psi_n(x)$ satisfying the equation $\mathcal{D}\psi_n = \lambda_n\psi_n$, the function $\tilde{\psi}_n = \gamma^5\psi_n$ is also an eigenfunction with the eigenvalue $\tilde{\lambda}_n = -\lambda_n$.

The idea is to use the spectral decomposition of the fermion Green's function with a small but non-zero quark mass

$$\langle q(x)\bar{q}(y) \rangle = \sum_n \frac{\psi_n(x)\psi_n^\dagger(y)}{i\lambda_n - m} \quad (5.1)$$

Set $x = y$ and integrate over d^4x . We have

$$V \langle \bar{q}q \rangle = -m \sum_{\lambda_n > 0} \frac{1}{\lambda_n^2 + m^2} \quad (5.2)$$

where the chiral symmetry of the spectrum has been used and the contribution of the zero modes $\lambda_n = 0$ has been neglected (it is justified when the volume V is large enough [26]). Perform the averaging over gauge fields and take *first* the limit $V \rightarrow \infty$ and *then* the limit $m \rightarrow 0$. The sum can be traded for the integral:

$$\langle \bar{q}q \rangle = -m \int \frac{\rho(\lambda)}{\lambda^2 + m^2} d\lambda = -\frac{1}{\pi} \rho(0) \quad (5.3)$$

The rightmost-hand-side of Eq.(5.3) is only the non-perturbative m -independent part of the condensate. There is also a perturbative ultraviolet-divergent piece $\propto m\Lambda_{ultr}^2$ which is proportional to the quark mass, is related to large eigenvalues λ and is of no concern for us here.

Thus, the non-perturbative part of the quark condensate which is the order parameter of the symmetry breaking is related to small eigenvalues of Euclidean Dirac operator. There

should be a lot of them — a characteristic spacing between levels is $\delta\lambda \sim 1/(| < \bar{q}q > |V)$ which is much less than the characteristic spacing $\delta\lambda \sim 1/L$ for free fermions.

The question is what is the physical reason for these small eigenvalues to appear. As far as we know, the first pioneer paper where a mechanism for generating small eigenvalues was proposed is Ref.[27] where small eigenvalues appeared as zero modes of monopole-like gauge field configurations. The disadvantage of this model is that the monopole configurations are static whereas it is natural to expect that characteristic gauge fields contributing to the Euclidean path integral at $T = 0$ are more or less symmetric in all four directions with no particular axis being singled out. The model of instanton-antiinstanton liquid formulated in [28] (see in particular Ref. [28b] where the mechanism for spontaneous chiral symmetry breaking was suggested) and developed later in [29] is much better in this respect.

The basic assumption of the model is that a characteristic gauge field contributing in QCD path integral is a medium of instantons and antiinstantons as shown in Fig.3. It is not a “gas” of Callan, Dashen, and Gross [30] because the interaction between instantons and antiinstantons bringing about a short-range correlations between instanton positions and orientations cannot be neglected. A “liquid” is a more proper term.

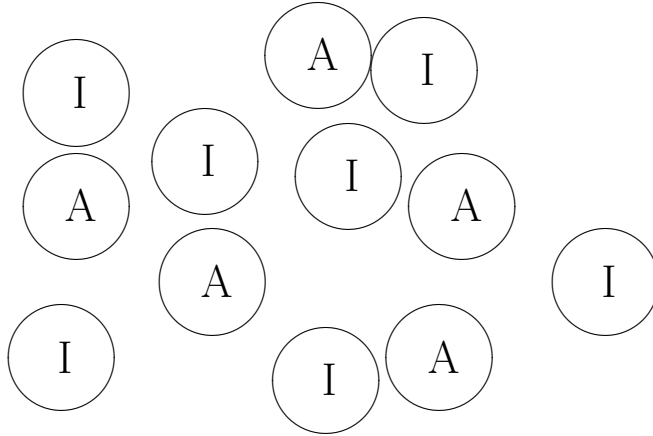


Fig. 3. Instanton-antiinstanton liquid.

Each individual instanton and antiinstanton involves a fermion zero mode [31]. Assuming the constant density of quasi-particles $\propto \mu_{had}^4$, the total number of zero modes in the Euclidean volume V is $N \sim V\mu_{had}^4$. However, these are not *exact* zero modes. They are shifted from zero due to interaction between instantons and antiinstantons (a nonzero overlap between individual instanton and antiinstanton zero modes. Assuming their uniform spreading in the range of eigenvalues $\Delta\lambda \sim \mu_{had}$, the volume density of *quasi-zero modes* is $\rho(0) \sim N/(V\Delta\lambda) \sim \mu_{had}^3$. Due to Eq.(5.3), a non-zero quark condensate appears ⁵.

⁵The assumption of quasi-uniform spreading of eigenvalues is not so innocent. It probably holds only in the theory with several light dynamical quarks, but not in the quenched theory ($N_f = 0$) where it is natural to expect a

This picture is rather similar to what happens in a doped superconductor with high enough doping. When a characteristic distance between individual atoms of the admixture is not large, the wave functions of outer electrons of these atoms overlap, and the electrons can jump from site to site. If the set of atoms of admixture with a noticeable overlap of wave functions forms a connected network in the space, the electrons can travel through this network at large distances and the specimen is a *conductor*. Note that it is not a standard metal mechanism of conductivity when the medium is a crystal, has the long-range order, and the electron wave functions are periodic Bloch waves. Here the distribution of the dope whose electrons are responsible for conductivity is stochastic and wave functions are complicated. The essential is that they are *delocalized*.

Thus, one can say that the vacuum of *QCD* is the “conductor” in a certain sense. For sure, there is no conductivity of anything in usual Minkowski space-time. Only the Euclidean vacuum functional has “conducting” properties. In principle, one can introduce formally the fifth time and write an analog of Kubo formula for conductivity in *QCD*, but the physical meaning of this “conductivity” is not clear. It is sufficient to say that, in a characteristic Euclidean gauge field background, the eigenfunctions of Dirac operator corresponding to small eigenvalues are delocalized.

What happens if we heat the system ? The effective coupling constant $g^2(T)$ decreases, the action of individual instantons $S = 8\pi^2/g^2(T)$ increases, and the density of quasi-particles $\propto \exp\{-S\}$ decreases.

Let us look first at the doped superconductor when we decrease the density of admixture. Below some critical density, the set of atoms with essential overlapping of wave functions does not form a connected network in 3-dimensional space anymore. The electrons can no longer travel far through this network, wave functions become localized, and the specimen is an insulator. This is called the percolation phase transition (see e.g. [33] for detailed discussion).

Likewise, there is a critical temperature in *QCD* above which instantons and antiinstantons do not form anymore a connected cluster with an essential overlap of individual fermion zero modes [what overlap is “essential” and what is not is a numerical question. For condensed matter systems (but not for *QCD* in this context) a computer estimates for the critical admixture density has been performed]. At high temperatures, few remaining quasi-particles tend to form “instanton-antiinstanton molecules” (See Fig.4). The individual zero modes are not *spread out* uniformly in the range $\Delta\lambda \sim \mu_{hadr}$ as is the case at zero temperature where instantons and antiinstantons form an infinite cluster, but are just *shifted* by the value $\sim \mu_{hadr}$ due to interaction in individual molecules. Small eigenvalues in the spectrum of Dirac operator are absent and the fermion condensate is zero [24].

singular behaviour of the spectral density near zero: $\rho(\lambda) \sim 1/\lambda$ so that the “fermion condensate” (i.e. the vacuum expectation value $\langle \bar{q}q \rangle_0$ where quark fields are treated as external sources) is infinite [32].

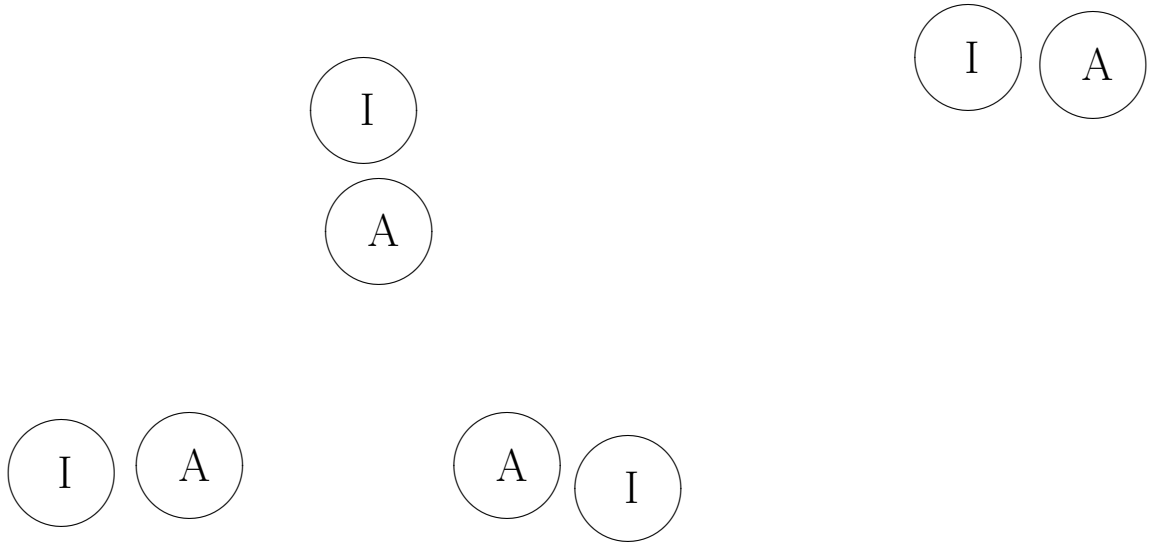


Fig. 4. Gas of instanton-antiinstanton molecules (high T).

Of course, this picture is too heuristic and qualitative. A serious quantitative study of fermion eigenvalues and eigenfunctions at non-zero temperature, and especially in the region $T \sim T_c$ has not yet been done. It is the task (a very interesting and important one) for future explorers.

It is worthwhile to emphasize once more that this scenario of percolation phase transition leading to the molecular high-temperature phase is expected to hold only at $N_f \geq 2$. For $N_f = 1$ with arbitrary small but nonzero fermion mass, molecules get ionized and the “medium” presents a very dilute instanton-antiinstanton gas — the instanton density involves a product of two small factors: $\exp\{-8\pi^2/g^2(T)\}$ and the fermion mass m . Differentiating $\log Z$ over m and sending m to zero, one gets a small but non-zero quark condensate [16, 17]. Cf. the analogous situation in the Schwinger model [34]

6 Disoriented chiral condensate.

When we talked in previous sections about “experimental” tests of theoretical predictions, we meant numerical lattice experiment. It is the unfortunate reality of our time that the feedback between theory and real laboratory experiment has drastically deteriorated: what is interesting from theoretical viewpoint cannot very often be measured in laboratory and what can be measured is not interesting.

However, speaking of the particular problem of the phase transition in QCD associated with chiral symmetry restoration, an intriguing possibility exists that a direct experimental evidence for such a transition can be obtained at the high-energy heavy ion collider RHIC

which is now under construction. After a head-on collision of two energetic heavy nuclei, a high temperature hadron “soup” is created.

We do not call this soup the quark-gluon plasma because, even at RHIC energies, the temperature would not be high enough to provide a sufficient smallness of the effective coupling $g^2(T)$ and to make the perturbation theory over this parameter meaningful. What is important, however, is that, at RHIC energies, the temperature of the soup would be well above the estimate (3.9) for the phase transition temperature. The high-temperature state created in heavy nuclei collision would exist for a very short time after which it expands, is cooled down and decays eventually into mesons.

Let us look in more details at the cooling stage. At high temperature, the fermion condensate is zero. Below phase transition, it is formed and breaks spontaneously chiral symmetry. This breaking means that the vacuum state is not invariant under the chiral transformations (3.2) and a direction in isotopic space is distinguished. What particular direction — is a matter of chance. This direction is specified by the condensate matrix

$$\Sigma_{ff'} = \langle \bar{q}_L f q_R f' \rangle \quad (6.1)$$

For simplicity, we have assumed up to now that

$$\Sigma_{ff'} = -\Sigma \delta_{ff'} \quad (6.2)$$

, but any unitary matrix can be substituted for $\delta_{ff'}$ (of course, it can be brought back in the form $\delta_{ff'}$ by a chiral transformation). In different regions of space, cooling occurs independently and directions of condensate are not correlated. As a result, domains with different directions of condensate shown in Fig.5 are formed (cf. cooling down of a ferromagnetic below the Curie point).

In our World, we do not observe any domains, however. The direction of the condensate in all spatial points is identical. This is a consequence of the fact that u - and d - quarks have non-zero masses which break chiral symmetry explicitly, the vacuum energy involves a term

$$E_{vac} \sim \text{Tr}\{\mathcal{M}^\dagger \Sigma\} + \text{c.c.} \quad (6.3)$$

and the only true vacuum state is (6.2) (in the basis where the quark mass matrix \mathcal{M} is diagonal).

However, the masses of u - and d - quarks are rather small and one can expect that the domains with “wrong” direction of the condensate are sufficiently developed during the cooling stage before they eventually decay into true vacuum (6.2) with emission of pions.⁶

This is a crucial assumption. A theoretic estimate of the characteristic size of domains they reach before decaying is very difficult and there is no unique opinion on this question in the

⁶Fig.5 implies the existence of several domains and describes better the physical situation immediately after the “phase crossover” in early Universe. Probably, the size of the hot fireball produced in collision of two nuclei is too small and the cooling occurs too fast for several domains to be developed. The popular “baked Alaska” scenario [35] implies the formation of only one domain with (generally) wrong flavour orientation.

literature. But if this assumption is true, we can expect to observe a very beautiful effect [35]. From the true vacuum viewpoint, a domain with disoriented $\Sigma_{ff'}$ is a classical object — kind of a “soliton” (parentheses are put because it is not stable) presenting a *coherent* superposition of many pions. The mass of this quasi-soliton is much larger than the pion mass. The existence of such multipion coherent states was discussed long ago in pioneer papers [36] but not in relation with thermal phase transition.

Eventually, these objects decay into pions. Some of the latter are neutral and some are charged. As all isotopic orientations of the condensate in the domains are equally probable, the *average* fractions of π^0 , π^+ , and π^- are equal: $\langle f_{\pi^0} \rangle = \langle f_{\pi^\pm} \rangle = \frac{1}{3}$ as is also the case for incoherent production of pions in, say, pp collisions where no thermalized high- T hadron soup is created.

But the *distribution* $P(f)$ over the fraction of, say, neutral pions is quite different in the case of incoherent and coherent production. In incoherent case, $P(f)$ is a very narrow Poissonic distribution with the central value $\langle f_{\pi^0} \rangle = 1/3$. The events with $f_{\pi^0} = 0$ or with $f_{\pi^0} = 1$ are highly improbable: $P(0) \sim P(1) \sim \exp\{-CN\}$ where $N \gg 1$ is the total number of pions produced.

For coherent production, the picture is quite different. $\Sigma_{ff'}$ is proportional to a $SU(2)$ matrix. Factorizing over $U(1)$, one can define a unit vector in isotopic space $\in S^2$. The fraction of π^0 produced would be just $f = \cos^2 \theta$ where θ is a polar angle on S^2 . The probability to have a particular polar angle θ normalized in the interval $0 \leq \theta \leq \pi/2$ [the angles $\theta > \pi/2$ do not bring about anything new as $f(\pi - \theta) = f(\theta)$] is $P(\theta) = \sin \theta$. After an elementary transformation, we get a normalized probability in terms of f :

$$P(f)df = \frac{df}{2\sqrt{f}} \quad (6.4)$$

As earlier, $\langle f \rangle = 1/3$, but the distribution in f is now wide and the values $f = 0$ and $f = 1$ are quite probable.

Thus, a hope exists that in, experiments with heavy ion collisions at RHIC, wild fluctuations in the fractions of neutral and charged pions would be observed. That would be a direct experimental indication that a quasi-phase-transition occurs where domains of disoriented chiral condensate of noticeable size are developed in a cooling stage. One can recall in this respect mysterious Centauro events with anomalously large fraction of neutral or of charged particles observed in cosmic ray experiments [37]. Who knows, may be that *was* the first experimental observation of the QCD phase transition.

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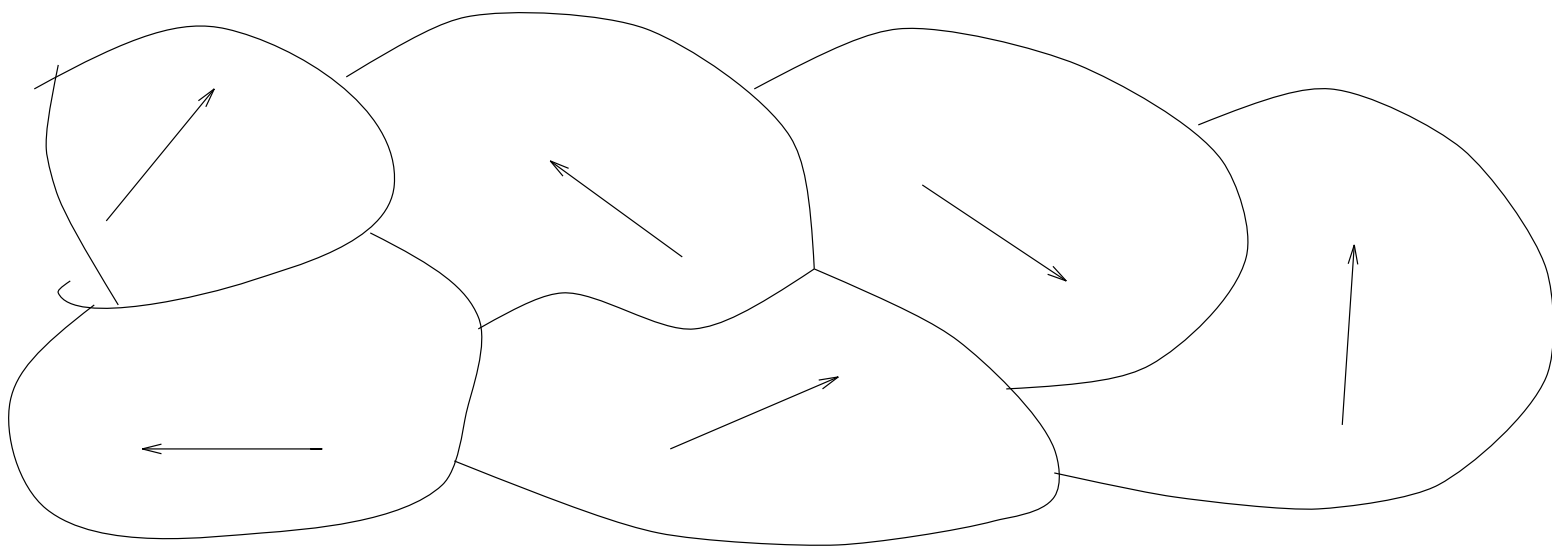
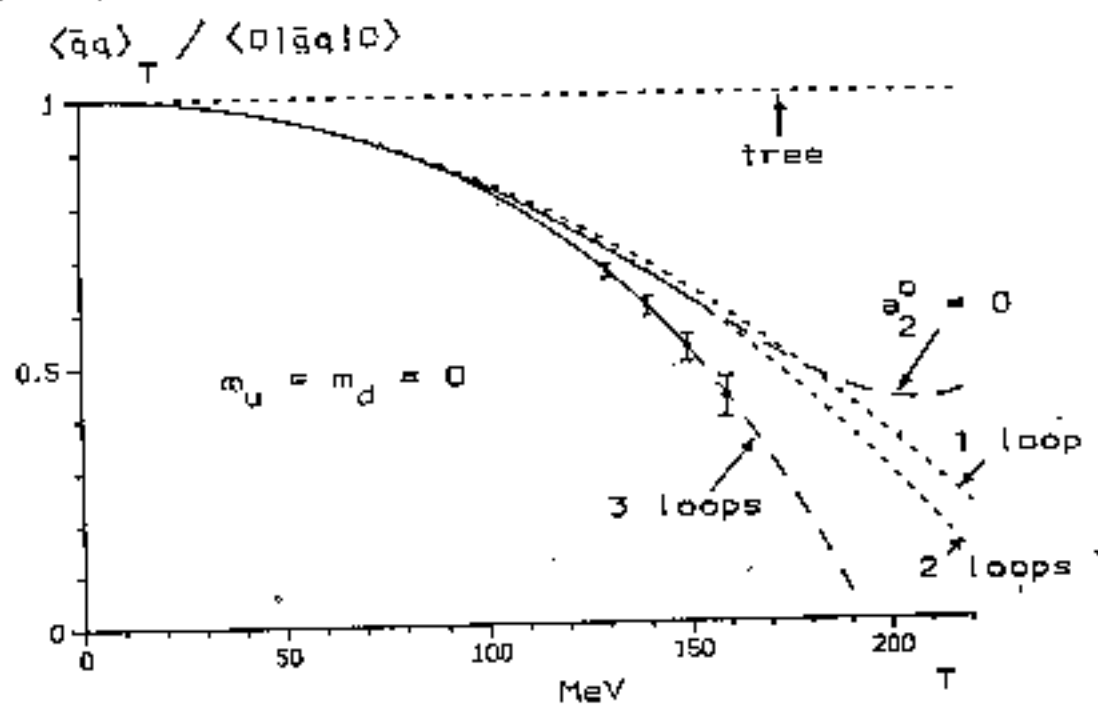


Fig. 5. Domains of disoriented chiral condensate.



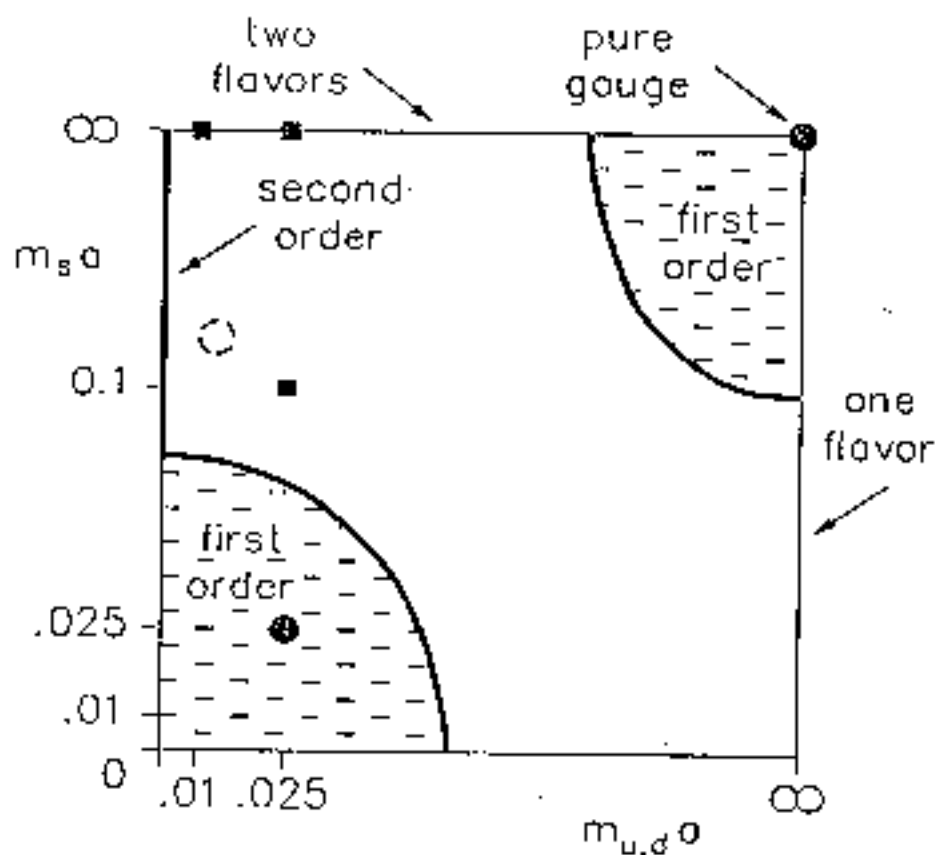


FIG. 2. Presence and absence of the finite-temperature QCD phase transition as a function of $m_{u,d}a$ and $m_s a$. Mass values for which the transition is and is not seen on a $16^3 \times 4$ lattice are denoted respectively by solid circles and squares. The physical point, indicated roughly by the dashed circle, lies in the region of no transition.